2. By Definition 4, $f_T(92,60) = \lim\limits_{h\to 0} \dfrac{f(92+h,60) - f(92,60)}{h}$, which we can approximate by considering $h = 2$ and $h = -2$ and using the values given in Table 1: $f_T(92,60) \approx \dfrac{f(94,60) - f(92,60)}{2} = \dfrac{111 - 105}{2} = 3$, $f_T(92,60) \approx \dfrac{f(90,60) - f(92,60)}{-2} = \dfrac{100-105}{-2} = 2.5$. Averaging these values, we estimate $f_T(92,60)$ to be approximately 2.75. Thus, when the actual temperature is 92 °F and the relative humidity is 60%, the apparent temperature rises by about 2.75 °F for every degree that the actual temperature rises.

Similarly, $f_H(92,60) = \lim\limits_{h\to 0} \dfrac{f(92,60+h) - f(92,60)}{h}$ which we can approximate by considering $h = 5$ and $h = -5$: $f_H(92,60) \approx \dfrac{f(92,65) - f(92,60)}{5} = \dfrac{108-105}{5} = 0.6$, $f_H(92,60) \approx \dfrac{f(92,55) - f(92,60)}{-5} = \dfrac{103-105}{-5} = 0.4$. Averaging these values, we estimate $f_H(92,60)$ to be approximately 0.5. Thus, when the actual temperature is 92 °F and the relative humidity is 60%, the apparent temperature rises by about 0.5 °F for every percent that the relative humidity increases.

6. (a) The graph of f decreases if we start at $(-1,2)$ and move in the positive x-direction, so $f_x(-1,2)$ is negative.

(b) The graph of f decreases if we start at $(-1,2)$ and move in the positive y-direction, so $f_y(-1,2)$ is negative.

(c) $f_{xx} = \frac{\partial}{\partial x}(f_x)$, so f_{xx} is the rate of change of f_x in the x-direction. f_x is negative at $(-1,2)$ and if we move in the positive x-direction, the surface becomes less steep. Thus the values of f_x are increasing and $f_{xx}(-1,2)$ is positive.

(d) f_{yy} is the rate of change of f_y in the y-direction. f_y is negative at $(-1,2)$ and if we move in the positive y-direction, the surface becomes steeper. Thus the values of f_y are decreasing, and $f_{yy}(-1,2)$ is negative.

8. $f_x(2,1)$ is the rate of change of f at $(2,1)$ in the x-direction. If we start at $(2,1)$, where $f(2,1)=10$, and move in the positive x-direction, we reach the next contour line (where $f(x,y)=12$) after approximately 0.6 units. This represents an average rate of change of about $\frac{2}{0.6}$. If we approach the point $(2,1)$ from the left (moving in the positive x-direction) the output values increase from 8 to 10 with an increase in x of approximately 0.9 units, corresponding to an average rate of change of $\frac{2}{0.9}$. A good estimate for $f_x(2,1)$ would be the average of these two, so $f_x(2,1)\approx 2.8$. Similarly, $f_y(2,1)$ is the rate of change of f at $(2,1)$ in the y-direction. If we approach $(2,1)$ from below, the output values decrease from 12 to 10 with a change in y of approximately 1 unit, corresponding to an average rate of change of -2. If we start at $(2,1)$ and move in the positive y-direction, the output values decrease from 10 to 8 after approximately 0.9 units, a rate of change of $\frac{-2}{0.9}$. Averaging these two results, we estimate $f_y(2,1)\approx -2.1$.

14. $f(x,y) = x^5 + 3x^3y^2 + 3xy^4 \quad \Rightarrow \quad f_x(x,y) = 5x^4 + 3 \cdot 3x^2 \cdot y^2 + 3 \cdot 1 \cdot y^4 = 5x^4 + 9x^2y^2 + 3y^4$,
$f_y(x,y) = 0 + 3x^3 \cdot 2y + 3x \cdot 4y^3 = 6x^3y + 12xy^3$.

18. $f(x, y) = x^y \quad \Rightarrow \quad f_x(x, y) = yx^{y-1}, \; f_y(x, y) = x^y \ln x$

26. $f(x,y,z) = x^2 e^{yz} \quad \Rightarrow \quad f_x(x,y,z) = 2xe^{yz}$, $f_y(x,y,z) = x^2 e^{yz}(z) = x^2 z e^{yz}$,

$f_z(x,y,z) = x^2 e^{yz}(y) = x^2 y e^{yz}$.

42. $xyz = \cos(x + y + z) \Rightarrow \dfrac{\partial}{\partial x}(xyz) = \dfrac{\partial}{\partial x}[\cos(x + y + z)] \Leftrightarrow$

$yz + xy\dfrac{\partial z}{\partial x} = [-\sin(x + y + z)]\left(1 + \dfrac{\partial z}{\partial x}\right)$, $[xy + \sin(x + y + z)]\dfrac{\partial z}{\partial x} = -[yz + \sin(x + y + z)]$, so

$\dfrac{\partial z}{\partial x} = -\dfrac{yz + \sin(x + y + z)}{xy + \sin(x + y + z)}$, $\dfrac{\partial}{\partial y}(xyz) = \dfrac{\partial}{\partial y}(\cos(x + y + z))$, and so by symmetry,

$\dfrac{\partial z}{\partial y} = -\dfrac{xz + \sin(x + y + z)}{xy + \sin(x + y + z)}$.

48. $f(x,y) = \ln(3x+5y) \quad \Rightarrow \quad f_x(x,y) = \dfrac{3}{3x+5y}$, $f_y(x,y) = \dfrac{5}{3x+5y}$. Then

$f_{xx}(x,y) = 3(-1)(3x+5y)^{-2}(3) = -\dfrac{9}{(3x+5y)^2}$, $f_{xy}(x,y) = -\dfrac{15}{(3x+5y)^2}$, $f_{yx}(x,y) = -\dfrac{15}{(3x+5y)^2}$,

and $f_{yy}(x,y) = -\dfrac{25}{(3x+5y)^2}$.